Polynomial

An expression of the form \( p(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n \) where \( a_n \neq 0 \) is called a polynomial in one variable \( x \) of degree \( n \), where; \( a_0, a_1, a_2, \ldots, a_n \) are constants and they are called the coefficients of \( x_0, x, x^2, \ldots, x^n \). Each power of \( x \) is a non-negative integer.

Eg: \(-2x^2 - 5x + 1\) is a polynomial of degree 2

Note: \( \sqrt{x} + 3 \) is not a polynomial

- A polynomial \( p(x) = ax + b \) of degree 1 is called a linear polynomial Eg: \( 5x - 3, 2x \) etc
- A polynomial \( p(x) = ax^2 + bx + c \) of degree 2 is called a quadratic polynomial Eg: \( 2x^2 + x - 1 \)
- A polynomial \( p(x) = ax^3 + bx^2 + cx + d \) of degree 3 is called a cubic polynomial.
  Eg: \( \sqrt{3}x^3 - x + \sqrt{5}, x^3 - 1 \) etc

Zeroes of a polynomial: A real number \( k \) is called a zero of polynomial \( p(x) \) if \( p(k) = 0 \). If the graph of \( y = p(x) \) intersects the X-axis at \( n \) times, the number of zeroes of \( y = p(x) \) is \( n \).

- A linear polynomial has only one zero.
- A quadratic polynomial has two zeroes.
- A cubic polynomial has three zeroes.

Graphs of different types of polynomials:

- Linear polynomial:- The graph of a linear polynomial \( ax + b \) is a straight line, intersecting X-axis at one point

- Quadratic polynomial:-
  (i) Graph of a quadratic polynomial \( p(x) = ax^2 + bx + c \) is a parabola open upwards like U, if \( a > 0 \) & intersects x-axis at maximum two distinct points.
(ii) Graph of a quadratic polynomial \( p(x)=ax^2 + bx + c \) is a parabola open downwards like \( \cap \) if \( a < 0 \) & intersects x-axis at maximum two distinct points.

- Cubic polynomial and its graph:- in general a polynomial \( p(x) \) of degree \( n \) crosses the x-axis at most \( n \) points.

For a quadratic polynomial:- If \( \alpha, \beta \) are zeroes of \( p(x) = ax^2 + bx + c \) then,

1. Sum of zeroes = \( \alpha + \beta = -\frac{b}{a} = \frac{-\text{coefficients of } x}{\text{coefficient of } x^2} \)
2. Product of zeroes= \( \alpha \cdot \beta = \frac{c}{a} = \frac{\text{constant term}}{\text{coefficient of } x^2} \)

- A quadratic polynomial whose zeroes are \( \alpha \) and \( \beta \), is given by:
  \[ p(x) = x^2 - (\alpha + \beta)x + \alpha \beta \]
- If \( \alpha, \beta \) and \( \gamma \) are zeroes of the cubic polynomial \( ax^3 + bx^2 + cx + d \) then:
\[
\begin{align*}
\alpha + \beta + \gamma &= -\frac{b}{ac} \\
\alpha\beta + \beta\gamma + \gamma\alpha &= -\frac{d}{a} \\
\alpha\beta\gamma &= -d
\end{align*}
\]

- If \(\alpha, \beta, \& \gamma\) are zeroes of a cubic polynomial \(p(x)\),
  \[p(x) = x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma\]

**Division algorithm for polynomials:** If \(p(x)\) and \(g(x)\) are any two polynomials with \(g(x) \neq 0\), then we have polynomials \(q(x)\) and \(r(x)\) such that
\[P(x) = g(x) \times q(x) + r(x),\] where \(r(x) = 0\) or degree of \(r(x) <\) degree of \(g(x)\).

**Nature of graph of polynomial**

\[P(x) = ax^2 + bx + c:\]

**Case-1** When polynomial \(ax^2 + bx + c\) is factorable in two distinct linear factors.

In this case, curve cuts X-axis at two distinct points. The co-ordinate of the vertex of parabola are \((-\frac{b}{2a}, -\frac{D}{2a})\) where \(D = b^2 - 4ac\). The x-coordinates of these points are the two zeroes of the polynomial.

\[
\begin{align*}
\text{(i) } a > 0
\end{align*}
\]

\[
\begin{align*}
\text{(ii) } a < 0
\end{align*}
\]

**Case 2:** When Polynomial \(ax^2 + bx + c\) is factorisable into two equal factors.

In this case, curve touches X-axis at the point \((-\frac{b}{2a}, 0\). The x- Co-ordinates of the point gives two equal zeroes of the polynomial.
Case- 3 When Polynomial $ax^2 + bx + c$ is not factorizable. In this case, the curve doesn’t cut or touches $X$-axis.

Level – I

1. Find the value of zeroes of the polynomials $p(x)$ as shown in the graph and hence find the polynomial. (CBSE 2014-15).

2. Let $\alpha$ and $\beta$ are the zeroes of a quadratic polynomial $2x^2 - 5x - 6$ then form a quadratic polynomial whose zeroes are $\alpha + \beta$ and $\alpha\beta$. (CBSE 2011)
3. Check whether $x^2 + 3x + 1$ is a factor of $3x^4 + 5x^3 - 7x^2 + 2x + 2$? (CBSE 2010)

4. Can $(x-7)$ be the remainder on division of a polynomial $p(x)$ by $(7x + 2)$? Justify your answer (CBSE 2010)

5. What must be subtracted from the polynomial $f(x) = x^4 + 2x^3 - 13x^2 - 12x + 21$, so that the resulting polynomial is exactly divisible by $x^2 - 4x + 3$? (CBSE 2013)

6. Write the degree of zero polynomial?

7. Find the zeroes of a quadratic polynomial $6x^2 - 7x - 3$ and verify the relationship between the zeroes and the coefficients? (CBSE 2014-15)

8. Find the quadratic polynomial sum of whose zeroes is $2√3$ and their product is $2$? (CBSE 2008)

**Level II**

9. If the sum of squares of the zeroes of the polynomials $6x^2 + x + k$ is $\frac{25}{36}$, find the value of k? (CBSE 2014-15)

10. If one zero of the quadratic polynomial $f(x)= 4x^2 - 8kx - 9$ is negative of the other, then find the value of $k$? (CBSE 2014-15)

11. Find the values of $k$ for which the quadratic equation $9x^2 - 3kx + k = 0$ has equal roots. (CBSE 2014)

12. On dividing $3x^3 - 2x^2 + 5x + 5$ by the polynomial $p(x)$, the quotient and remainder are $x^2 - x + 2$ and $-7$ respectively. Find $p(x)$? (CBSE 2013)

13. Find all the zeroes of the polynomial $x^4 + x^3 - 9x^2 - 3x + 18$, if two of its zeroes are $\sqrt{3}$ and $\sqrt{-3}$. (CBSE 2010,13)

14. If $\alpha , \beta$ are zeroes of the quadratic polynomial $p(x) = x^2 - (k - 6)x + (2k + 1)$. Find the value of $k$ if $\alpha + \beta = \alpha \beta$. (CBSE 2010)

15. If the zeroes of the polynomial $x^2 - 5x + k$ are the reciprocal of each other, then find the value of $k$? (CBSE 2011)

16. If $\alpha$ and $\beta$ are zeroes of the quadratic polynomial $x^2 - 6x + a$, find the value of $a'$. If $3\alpha + 2\beta = 20$. (CBSE 2010)
17. On dividing \(3x^3 + 4x^2 + 5x - 13\) by a polynomial \(g(x)\), the quotient and remainder are \(3x + 10\) and \(16x - 43\) respectively. Find the polynomial \(g(x)\). (CBSE 14-15)

18. If -5 is a root of quadratic equation \(2x^2 + px - 15 = 0\) and the quadratic equation \(p(x^2 + x)k = 0\) has equal roots, find the value of \(k\). (CBSE 2106)

19. If \(\alpha, \beta\) and \(\gamma\) are zeroes of the polynomial \(6x^3 + 3x^2 - 5x + 1\), then find the values of \(\alpha^{-1} + \beta^{-1} + \gamma^{-1}\). (CBSE 2010)

20. Form a cubic polynomial whose zeroes are 3, 2 and -1. Hence find
   (i) Sum of its zeroes
   (ii) Sum of the product, taken two at a time
   (iii) Product of its zero.

(SELF EVALUATION QUESTIONS)

21. Find the number of zeroes of \(p(x)\) in each case, for some polynomials \(p(x)\).

22. If \(\alpha\) and \(\beta\) are the zeroes of the equation \(6x^2 + x - 2 = 0\), find \(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\)

23. If one of the zeroes of the polynomial \(2x^2 + px + 4 = 0\) is 2, find the other zero, also find the value of \(p\)

24. If one zero of the polynomial \((\alpha^2 + 9)x^2 + 13x + 6\alpha\) is reciprocal of the other. Find the value of \(a\). (All India)
Value Based Questions

25. If $\alpha$ be the number of person who take junk food, $\beta$ be the person who take food at home and $\alpha$ and $\beta$ be the zeroes of quadratic polynomial $f(x) = x^2 - 3x + 2$, then find a quadratic polynomial whose zeroes are $\frac{1}{2\alpha + \beta}$ and $\frac{1}{2\beta + \alpha}$, which way of taking food you prefer and why?

26. If the number of apples and mangoes are the zeroes of the polynomial $3x^2 = 8x - 2k + 1$ and the number of apples is 7 times the number of mangoes, then find the number of zeroes and value of $k$. What are benefits of fruits in our daily life?